Instructions: Welcome to quals! First, put your name on the paper you turn in (ideally a Blue Book). There are 4 problems on this exam, each of which is worth 25 points. The exam is closed notes, closed book. Calculators may be used. Good luck!

Problem 1: Consider the combined feedforward/feedback control structure shown in Figure 1.

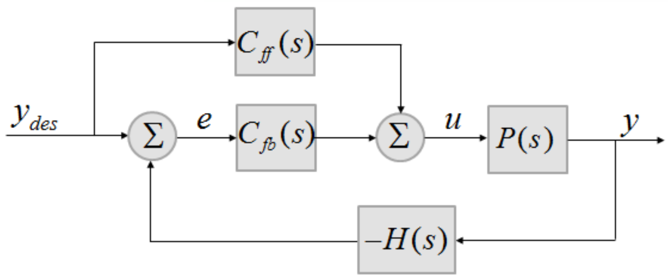

Figure 1: Block diagram of the feedforward/feedback control structure for problem 1.

Suppose that the constituent transfer functions are given by:

$$C_{ff}(s) = K_{ff}, \quad (1)$$
$$C_{fb}(s) = K_{fb}, \quad (2)$$
$$P(s) = \frac{1}{1.5s+1}, \quad (3)$$
$$H(s) = \frac{1}{0.5s+1}, \quad (4)$$

where K_{ff} and K_{fb} are positive scalar values (gains).

a) (15 points) For what (positive) values of K_{ff} and K_{fb} is the closed-loop system stable? Your answer should indicate a range of K_{ff} and K_{fb} for which the system is stable. If you think the system is stable for all or no values of K_{ff} or K_{fb}, you may just say "stable for all values" or "stable for no values."

b) (10 points) Assuming $K_{fb} = 8$, what value of K_{ff} is required to achieve a DC gain of 1 from y_{des} to y?

Problem 2: The longitudinal dynamics of a luxury cruise ship (which are similar to the longitudinal dynamics of, say, a Carnival Cruise Lines non-luxury ship, except when the Carnival ship runs aground) are described by the following equations:

$$\tau_{eng}\dot{F}_{th} = F_{cmd} - F_{th}, \quad (5)$$
$$m\dot{v} = F_{th} - F_{drag}, \quad (6)$$
$$F_{drag} = k_{aero}(v + v_{wind})^2 + k_{hydro}(v + v_{cur})^2, \quad (7)$$

where the variables are defined as follows:

$F_{th} =$ Thrust force (N)

$F_{cmd} =$ Commanded thrust force (N)

$\tau_{eng} =$ Engine time constant (s)

$m =$ Ship mass (including "added mass" of displaced fluid) (kg)

$F_{drag} =$ Retarding force from aerodynamic and hydrodynamic drag (N)

$v =$ Ship speed ($\frac{m}{s}$)

$v_{wind} =$ Wind speed ($\frac{m}{s}$) - Positive denotes a headwind

$v_{cur} =$ Current speed ($\frac{m}{s}$) - Positive denotes current flowing against the ship

$k_{aero} =$ Lumped aerodynamic drag coefficient ($\frac{Ns^2}{m^2}$)

$k_{hydro} =$ Lumped hydrodynamic drag coefficient ($\frac{Ns^2}{m^2}$)

a) (15 points) Derive a linear approximation of the aforementioned longitudinal dynamic equations, around the equilibrium point $v_0 = 10\frac{m}{s}$, $v_{wind,0} = 5\frac{m}{s}$, $v_{cur,0} = 0\frac{m}{s}$, $F_{th,0} = 42,500N$, $F_{cmd,0} = 42,500N$, where the constant parameters are given by:

$\tau_{eng} = 10\ s$

$m = 4 \cdot 10^7\ kg$

$k_{aero} = 100\ \frac{Ns^2}{m^2}$

$k_{hydro} = 200\ \frac{Ns^2}{m^2}$

b) (10 points) Derive the transfer functions from δF_{cmd}, δv_{wind} ,and δv_{cur} to δv for the linearized model you derived in (a).

Problem 3: Consider the two control system configurations in Figure 2, where y_{des} represents a setpoint, d represents an external disturbance, and n represents sensor noise. For each of the following three sets of circumstances and control requirements, indicate which of the two control structures you would recommend. *Answers should be supported by mathematical analyses to receive full credit.*

a) (10 points) Suppose that the plant dynamics are not known precisely, but it is known that $0.5 \leq a \leq 1.5$. Furthermore, assume that the closed-loop performance requirements are:

- Stable closed-loop system;
- DC (steady-state) gain of 1 from y_{des} to y.

b) (10 points) Suppose that the plant dynamics are known precisely, with $a = 1$, and that the new closed-loop performance requirements are:

- Stable closed-loop system;
- DC (steady-state) gain of 1 from y_{des} to y;
- DC (steady-state) gain of 0 from d to y.

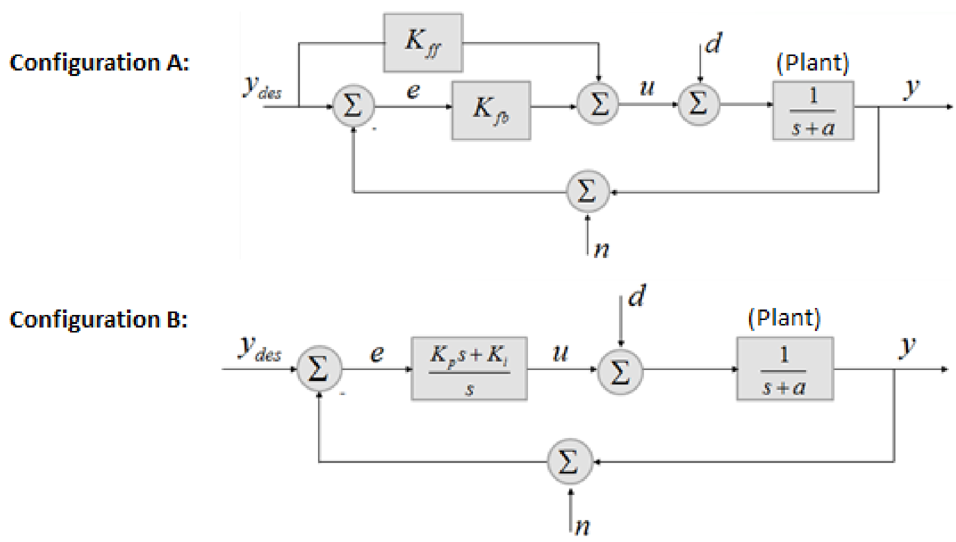

Figure 2: Block diagrams for the configurations to be analyzed in problem 3. Note: the negative sign by the summation junction indicates negative feedback.

c) (5 points) Suppose that the plant dynamics are known precisely, with $a = 1$, and that the new control requirements are as follows:

- Stable closed-loop system;
- DC (steady-state) gain of 1 from y_{des} to y;
- DC (steady-state) gain of 0.5 or less from $-n$ to y.

Problem 4: Consider the block diagram of Fig. 3, where a scalar gain (K) and first order filter (with time constant τ) are cascaded with a linear mystery system having a transfer function $G(s)$, whose Bode plot is given in Fig. 4.

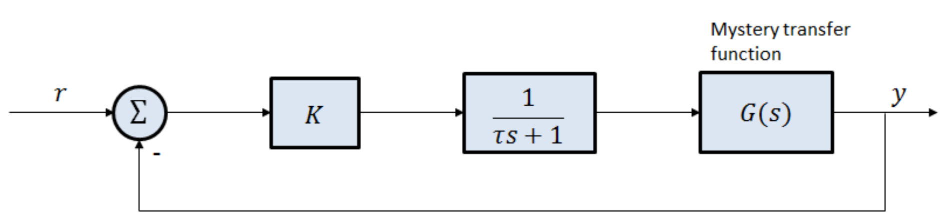

Figure 3: Block diagram for problem 4.

a) (15 points) Suppose that $\tau = 0.00001$. What is the maximum gain, K, for which the closed-loop system is stable?

b) (10 points) Suppose that $K = 1$. As the filter time constant, τ, is increased, does the system's phase margin increase, decrease, or stay the same?

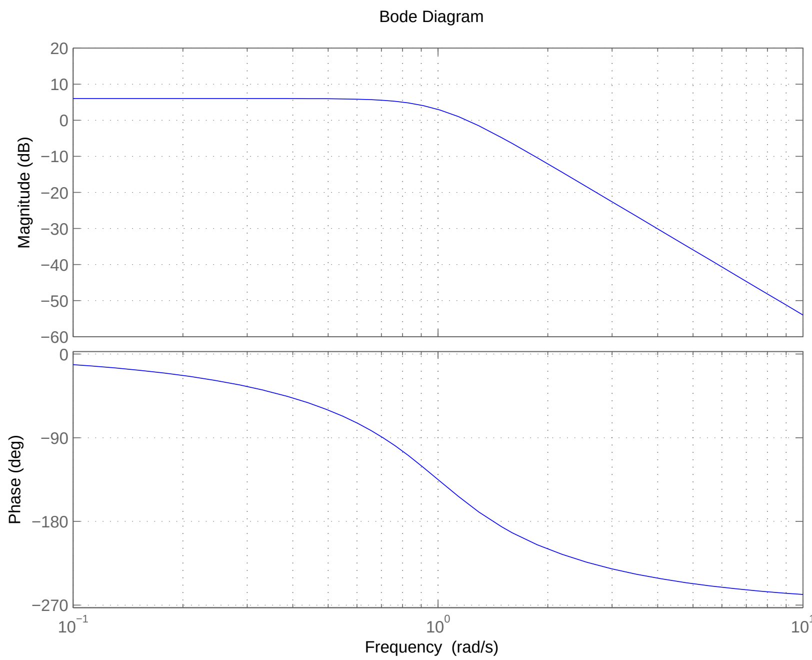

Figure 4: Bode plot of $G(s)$ for problem 4.