Homework 9

1. In the class, we classified second-order partial differential equations using two different definitions. Show that these definitions are equivalent to each other.

2. Consider the spring-mass-damper system

$$\frac{d^2X}{d\tau^2} + \epsilon\frac{dX}{d\tau} + X = 0$$

with $X(0) = \alpha = 0.5$ and $\dot{X}(0) = \beta = 0$.

(a) Obtain an exact solution to the above equation.

(b) Next, obtain an approximate solution by assuming that $X(\tau) = X_0(\tau) + \epsilon X_1(\tau) + \epsilon^2 X_2(\tau) + \cdots$. Substitute this expansion into the differential equation and the initial conditions. By comparing the coefficients of ϵ, obtain the differential equations and the corresponding initial conditions for the leading-order solution $X_0(\tau)$, the first-order correction $X_1(\tau)$ and the second-order correction $X_2(\tau)$.

(c) Solve these three differential equations.

(d) Assume $\epsilon = 0.05$. Plot the exact solution along with the solutions $X_0(\tau)$, $X_0(\tau) + \epsilon X_1(\tau)$ and $X_0(\tau) + \epsilon X_1(\tau) + \epsilon^2 X_2(\tau)$ in the same plot window.

(e) Repeat the above step for $\epsilon = 0.5$.

(f) Compare the two sets of results and comment on the accuracy of the asymptotic solutions.

For plotting the results, take $0 \leq t \leq 10$.