

1. Find the power series solutions of the following differential equations about  $x_0 = 0$ :

(a)  $(x^2 - 1)y'' + xy' - y = 0$ .

(b)  $y'' + e^x y' - y = 0$ .

You will need to determine if  $x=0$  is a regular point or not. For part (b), first expand  $e^x$  about  $x_0$  as a power-series. Plot the two solutions for each of the above, by retaining  $m$  terms with  $m = 1, 2, 5, 10, 20$  and  $50$ .

2. Consider the expression

$$\sum_{n=2}^{\infty} n(n-1)c_n x^n + 5 \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} + 2 \sum_{n=1}^{\infty} n c_n x^n.$$

Rewrite the above so that it is a single power series with a general term  $x^m$ .