Homework 6

1. Find the intervals on which the following differential equations are normal:
   (a) $y'' + 7xy' - 11y = \ln \sin \pi x$,
   (b) $\sqrt{x(1-x)}y''' - e^{-x} \sin xy' + y = 2 - x$,
   (c) $x^2 y''' - 3xy'' + 4y = \sinh x$.

2. Consider the differential equation

$$y'' + 4y = 4 \text{ on } (-\infty, \infty)$$

   with $y(0) = 2$ and $y'(0) = 0$. Given that both $1 + \cos 2x$ and $2\cos^2 x$ satisfy this differential equations and the initial conditions, what argument would you use to conclude that the solutions are equal, i.e.,

$$1 + \cos 2x = 2\cos^2 x.$$

3. Determine if the following set of functions are linearly dependent or independent. If they are linearly dependent, provide a relationship that shows the dependence.
   (a) $\{e^x, x, \cosh x\}$ on $(-\infty, \infty)$,
   (b) $\{e^x, e^{2x}\}$ on $(-\infty, \infty)$,
   (c) $\{x^2 - 1, x^2 + x + 1, x^2 + 3x + 5\}$ on $(-\infty, \infty)$.

4. During the lecture, we remarked on the superposition principle for the particular solutions of non-homogeneous linear ordinary differential equations. Use this principle to obtain a particular solution to

$$y'' - 6y' + 5y = -10x^2 - 6x + 32 + e^{2x}$$

   given that $3e^{2x}$ and $x^2 + 3x$ are respectively particular solutions of

$$y'' - 6y' + 5y = -9e^{2x} \text{ and } y'' - 6y' + 5y = 5x^2 + 3x - 16.$$