1. A mass m is subjected to a force $F(t, v)$ where t is the time and $v(t)$ is the velocity of the mass. It also experiences a resistive force $\eta v(t)$ where η is a constant. The motion of the mass is governed by the differential equation

$$m\frac{dv}{dt} = F - \eta v.$$

   Assuming that $F(t) = \alpha e^{-\beta t} v^3$, obtain a solution for $v(t)$. Assume that the initial velocity is v_0.

   Plot the direction fields and four solution curves for each of the following three sets of data:

   (a) $m = 10$, $\eta = 0$, $\alpha = 0.1$, $\beta = 0.1$,

   (b) $m = 10$, $\eta = 0.5$, $\alpha = 0.1$, $\beta = 0.1$,

   (c) $m = 10$, $\eta = 5.0$, $\alpha = 0.1$, $\beta = 0.1$.

   For the solution curves, take $v_0 = 0.5, 1, 2.0$ and 2.5.

   Interpret the solutions: What will the solution tend to as $t \to \infty$ in the presence and absence of the resistive force? Will the force be resistive if $\eta < 0$? Is there any resistance to motion if $\eta = 0$?

2. Obtain the general solution $y(x)$ for the homogeneous differential equation

$$(5x^2 - 2y^2)dx - xydy = 0.$$

3. The differential equation

$$\frac{dT}{dt} = k(T^4 - T_\infty^4)$$

   governs the temperature of a lumped mass cooling or heating due to radiation.

   (a) Solve the above equation for $T(t)$ assuming that $T(0) = T_0$.

   (b) Show that the differential equation takes the same form as the differential equation for lumped heat transfer by convection (discussed in the class) when $|T - T_\infty|$ is small compared to T_∞.