1. Consider the first-order linear differential equation

$$y' + p(x)y = q(x).$$

Show that $e^{\int p(x)dx}dx$ is an integrating factor for the above equation.

2. Consider the equation

$$\dot{P}(t) = P(t)(\alpha - \beta P(t)) - H.$$

In the above, $P(t)$ is the population of a species at time t, $\dot{P}$ is the time-derivative of $P(t)$ and H is a constant denoting the quantity of species removed from $P(t)$ per unit time. So for example, if $P(t)$ denotes the population of a species of fish, H could denote the quantity of fish harvested per year.

Assume that $\alpha = 2$ and $\beta = 0.2$. Plot the direction fields, phase portraits and solution curves for the following three cases:

(a) $H = 0.5$. Show also a solution curve for each of the regions separated by the critical points.

(b) $H = 5$. Show also a solution curve for each of the regions separated by the critical points.

(c) $H = 6$. Show also a solution curve for each of the regions separated by the critical points.

Interpret your results. Provide the plots as hard-copies. Create a hw4 folder on the shared folder on google drive and upload your code (maple or matlab) to it.

3. Consider the various regions defined the critical points of part (a) of Problem 2. The solution curves in each region stay in that particular region. Can you explain the reason for this? The solution curve in each region is either increasing or decreasing. Why can't there be any peaks or valleys in these curves?

4. Draw the direction fields for the following three differential equations.

(a) $y' = k(\alpha - y)(\beta - y), \alpha = 0.02, \beta = 0.05,$ and $k = 1.2$.

(b) $y' = cos(x + y)$.

(c) $y' = xy(1 + y)^{-1}$.

For the last two problems, use the range $-8 \leq x, y \leq 8$.