Homework 11

1. Consider the one-dimensional heat equation given by:

$$\frac{\partial^2 u}{\partial x^2} + Q(x,t) = \frac{1}{\alpha}\frac{\partial u}{\partial t}$$

with $u(0,t) = 50°\text{C}$ and $u(l,t) = 100°\text{C}$. Let the initial condition be $u(x,0) = 100°$ C. Solve for the temperature in the bar.

2. Assuming that the material is made of steel, plot the temperature distribution in the bar as a function of time for $t = 0, 5, 100, 1000$ seconds. In a separate plot window, plot the heat flux for the same times.

3. The vibrations of a uniform string of length l subjected to a frictional force are governed by

$$\frac{\partial^2 u}{\partial x^2} + 2\xi\frac{\partial u}{\partial t} = \frac{1}{C_b^2}\frac{\partial^2 u}{\partial t^2}.$$

Note that the second term on the left in the above equation is due to friction. We take $\xi > 0$. The string is fixed at both the ends and is given an initial displacement of $f(x)$ and zero initial velocity. Obtain a solution for the motion of the string using the separation of variables method. Also, determine the frequencies that are overdamped and the frequencies that are underdamped.