<u>Fluid Mechanics Qualifying</u>

<u>Examination</u>

<u>Sample Exam 2</u>

Allotted Time: 3 Hours

The exam is closed book and closed notes. Students are allowed one (double-sided) formula sheet.

There are <u>five</u> questions on this exam. Answer any <u>four</u> (each for 25 points).

If you answer all five, the best four will be considered.

State all your assumptions, and explain your reasoning clearly.

1.

a) Show that for an irrotational flow that is also incompressible, the velocity potential ϕ also satisfies the Laplace equation.

b) Furthermore, in two-dimensional, incompressible flows, a stream-function $\psi(x,y)$ exists such that

$$\frac{\partial \psi}{\partial x} = -v \qquad \frac{\partial \psi}{\partial y} = u$$

so that ϕ and ψ satisfy the Cauchy-Riemann equations:

$$\phi(x,y) + i\psi(x,y) = w(z)$$

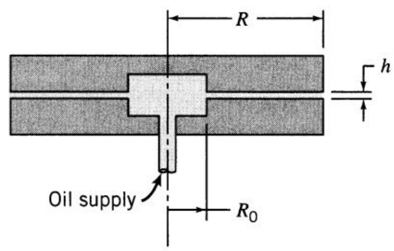
where *w(z)* is a complex analytic function of *z = x+iy* . Interpret <u>any four</u> of the following flows, write down expressions for the velocity components, and draw streamlines where possible:

I. $w(z) = z$
II. $w(z) = z^2$
III. $w(z) = m \log(z)$
IV. $w(z) = i \log(z)$
V. $w(z) = U a (z/a + a/z)$, with real positive constants U and a.

Hint: Feel free to use polar coordinates $z = r \exp(i\theta)$ for some of the above cases to simplify analyses.

2. Viscous liquid, at a volume flow rate of Q, is pumped through the central opening into the narrow gap between the parallel disks shown. The flow rate is low, so the flow is laminar, and the pressure gradient due to convective acceleration in the gap is negligible compared with the gradient caused by viscous forces (creeping flow). Obtain a general expression for the variation of average velocity in the gap between the disks. Note that for creeping flow, the velocity profile at any cross section in the gap is the same as for fully developed flow between stationary parallel plates. Evaluate the pressure gradient, dp/dr, as a function of the radius. Obtain an expression for p(r). Show that the net force required to hold the upper plate in the position shown is

$$F = \frac{3\mu Q R^2}{h^3}\left[1 - \left(\frac{R_0}{R}\right)^2\right]$$



3. Consider steady adiabatic $1D$ **compressible** flow of air through a variable area duct. At a certain section of the duct, the flow area is 0.2 m^2, the pressure is 80 kPa, the temperature is 5^0 C and the velocity is 200 m/s. If, at this section, the duct area is changing at a rate of 0.3 m^2/m (i.e., $dA/dx =$ 0.3 m^2/m), find dp/dx, dV/dx, and $d\rho/dx$. Assume a frictionless flow.

   Assume, for air, $R = 0.287$ kJ/kg·K and C_p = 1.004 kJ/kg·K.

4. Air is induced in a smooth insulated tube of 7.16 mm diameter by a vacuum pump. Air is drawn from a room, where the pressure and temperature are 101 kPa (abs) and 23^{0}C respectively, through a smoothly contoured converging nozzle. At section 1, where the nozzle joins the constant-area tube, the static pressure is 98.5 kPa (abs). At section 2, located some distance downstream in the constant-area tube, the temperature is 14^{0}C. Assume $k = 1.4$. Determine the following quantities at section 2:

   a. the mass flow rate

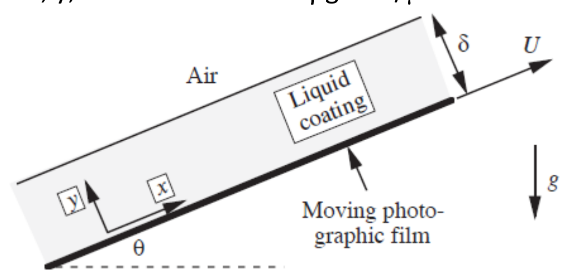
   b. the local isentropic stagnation pressure

   c. the friction force on the duct wall between sections 1 and 2

   What additional information and assumptions would you need if you are asked to determine the distance between the sections 1 and 2? Where would you find these additional quantities?

   Show the steps you would have followed to find the distance between the sections 1 and 2. Use either a symbol or a logical value for any missing quantity (for this question only).

5. The figure below shows a coating experiment involving a flat photographic film that is being pulled up from a processing bath by rollers with a steady velocity U at an angle θ to the horizontal. As the film leaves the bath, it entrains some liquid, and in this particular experiment it has reached the stage where: (a) the velocity of the liquid in contact with the film is $v_x = U$ at $y = 0$, (b) the thickness of the liquid is constant at a value δ, and (c) there is no net flow of liquid (as much is being pulled up by the film as is falling back by gravity).
    a. Write down the differential mass balance and simplify it.
    b. Write down the differential momentum balances in the x and y directions. Simply the momentum balance as much as possible.
    c. From the simplified y-momentum balance, derive an expression for the pressure p as a function of y, p, δ, g, and θ
    d. From the simplified x momentum balance, assuming that the air exerts a negligible shear stress τ_{yx} on the surface of the liquid at $y = \delta$, derive an expression for the liquid velocity v_x as a function of U, y, δ and α where $\alpha = \rho g \sin\theta/\mu$.



I. Assume the flow is steady and Newtonian with constant density ρ and viscosity μ.
II. The z direction, normal to the plane of the diagram may be disregarded entirely. Thus, not only is v_z zero, but all derivatives with respect to z are also zero.
III. Assume $v_y = v_z = 0$.
IV. Gravity acts vertically downward.
V. Assume that the pressure at the free surface between the air and the liquid is zero, and that the air pressure is uniformly zero everywhere. Also, assume $\partial p/\partial x = 0$.