PhD Qualifier: Dynamics Spring 2023

The duration of this exam is 3 hours. There are 5 questions and the students must work 4 of them. If more than 4 questions are attempted, the student must specify which 4 are be graded. If not specified, the first 4 questions will be graded. Each question is 25 points and you need 70 points to pass this exam. Good Luck!

1	2	3	4	5

Problem 1. For the slider-crank mechanism of Figure 1 at the instant shown the crank is horizontal and rotating at an angular speed ω of 600 rad·s^{-1} in the direction shown. Give the parameters L_1 = 0.08 m, L_2 = 0.2 m, and h = 0.04 m

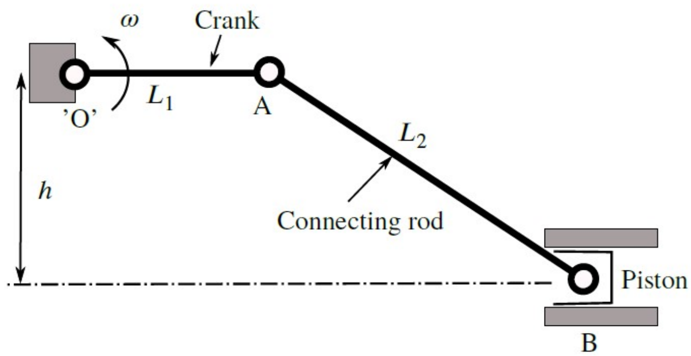

Fig. 1: Slider-crank mechanism

a) Determine the instantaneous center of zero velocity for the link AB.

b) Determine the angular speed of the connecting rod and the velocity of the piston.

Problem 2. A car is towing a caravan weighing 800 kg and is accelerating at 2 m·s^{-2} to the right, see Figure 2. For the parameters L_1 = 0.5 m, L_2 = 3 m, and h_1 = 1.4 m, and h_2 = 1 m;

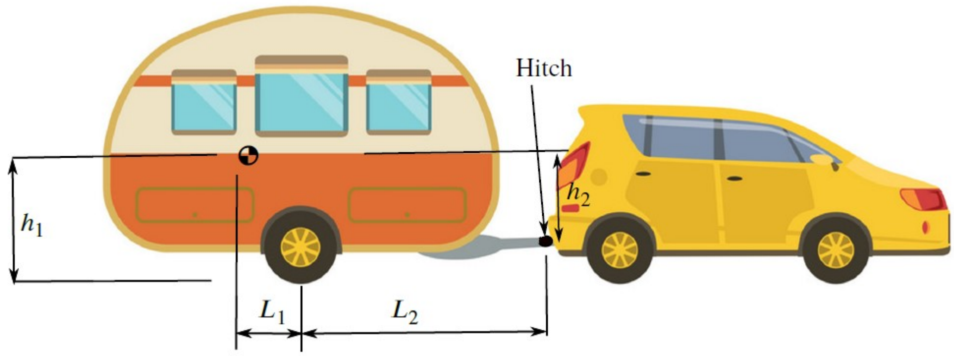

Fig. 2: Car towing a caravan along a horizontal road

a) Draw the free body diagram for the caravan.

b) Determine force applied to the hitch.

Hint: This problem is probably best solved by considering moments about the center of gravity of the caravan.

Problem 3. A ball with mass μ is attached to one end of a massless cord with length λ. You hold the other end of the cord over your head and swing the ball around yourself as shown:

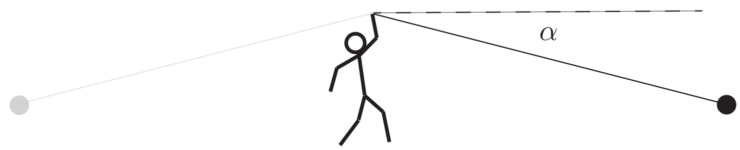

Fig. 3: Figure for Problem 3

You live in a universe devoid of friction, so once you get the ball going, you're able to hold your end of the cord at a fixed location while the ball continues to describe a circular arc around you, moving with constant speed and maintaining a constant angle α with respect to the horizontal plane containing your hand.

a) In terms of μ, λ, α, and/or the gravitational constant g, what's the tension in the cord? How much time does it take for the ball to complete each orbit around you?

b) To verify the plausibility of your second answer, compute the approximate numerical value of the orbital period — say, to the nearest second — for the case in which $m = 1$ kilogram, $\lambda = 4$ meters, $\alpha = \pi/12$ radians (as shown in the figure), and $g = 10$ meters per square second.

Problem 4. A block with mass m is joined to a block with mass M by a linear spring with stiffness k. The system is floating at rest in space, with the spring relaxed, when suddenly — at time $t = 0$ — a force with magnitude $F\delta(t)$, where F is a constant and $\delta(t)$ is the unit impulse function, is applied to the block with mass M along the axis of the spring, as shown:

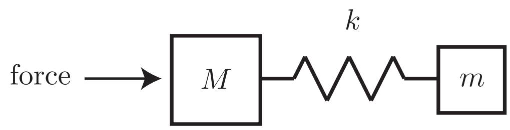

Fig. 4: Figure for Problem 4

a) The distance between the blocks will oscillate thereafter. In terms of M, m, and/or k, what's the frequency of oscillation? You can assume that the blocks move only along the axis of the spring.

b) For a given k and a given total mass $M + m$, is this frequency of oscillation maximized, minimized, or neither when $m = M$?

c) Now suppose that some linear damping is introduced that impedes the translation of the block with mass M, and that the external force has magnitude $F \sin \Omega t$ for some constant frequency Ω. In terms of Ω, M, and/or m, what value of k will minimize the steady-state oscillation of the block with mass M?

Problem 5. Circle the best answer. Each Sub-question is worth 5 points.

5.1. Which second-order system has a natural frequency of 2 rad/s and a damping ratio of 0.5?

[A] $2\ddot{x} + \dot{x} + 0.5x = 0$
[B] $4\ddot{x} + 4\dot{x} + x = 0$
[C] $\ddot{x} + 2\dot{x} + 4x = 0$
[D] $\ddot{x} + 0.5\dot{x} + 2x = 0$

5.2. A mass-spring-damper system has a spring of stiffness $1,000$ N/m, a mass of 10 kg, and a damping coefficient of 10 N/(m/s). What is the natural frequency of the system?

[A] $10/\sqrt{2}$ rad/s
[B] $10\sqrt{0.75}$ rad/s
[C] 10 rad/s
[D] 100 rad/s

5.3. What is the equation of motion for the following system with $k_1 = k_2 = b_1 = b_2 = 1$?

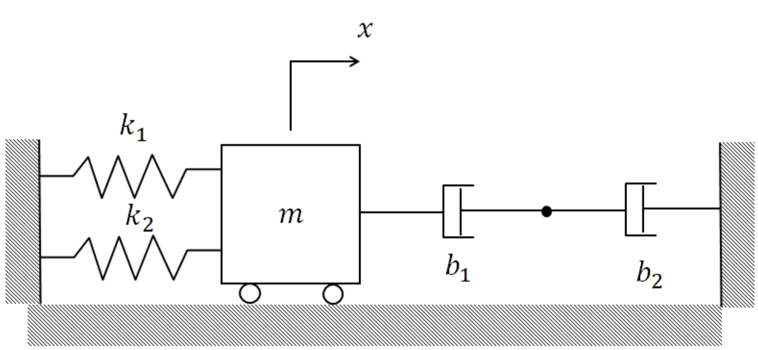

Fig. 5: Figure of Problem 5.4

[A] $\ddot{x} + 2\dot{x} + 2x = 0$
[B] $\ddot{x} + 0.5\dot{x} + 2x = 0$
[C] $\ddot{x} + 2\dot{x} + 0.5x = 0$
[D] $\dot{x} + 0.5x = 0$

5.4 Suppose the system shown below starts from rest and is driven by a sinusoidal force input $u(t)$. How long does it take (approximately) for the transients to decay and the system response $x(t)$ to reach a steady-state sinusoidal output?

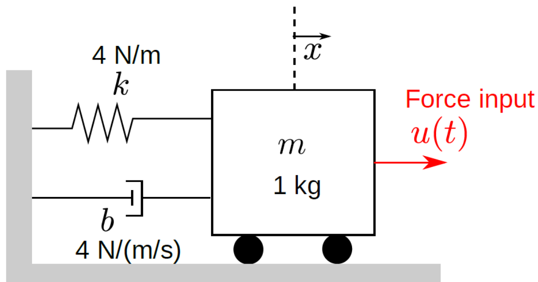

Fig. 6: Figure of Problem 5.5

$$
\begin{aligned}
&[A] && \approx 0.5sec. \\
&[B] && \approx 1sec. \\
&[C] && \approx 2sec. \\
&[D] && \approx 4sec. \\
&[E] && \approx 16sec.
\end{aligned}
$$

5.5. Consider the responses below of a homogeneous second-order system from the initial condition $x(0) = 4$ and $\dot{x}(0) = 0$.

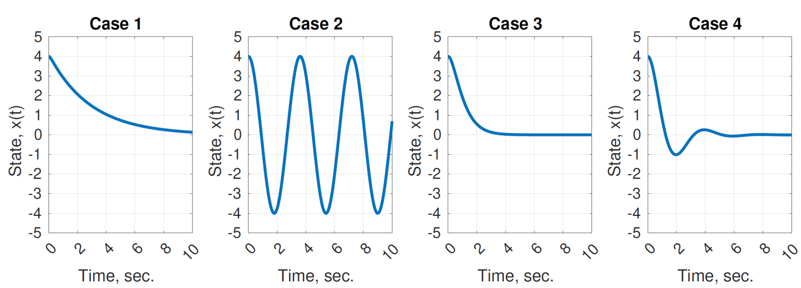

Fig. 7: Four Cases of Problem 5.5

Which of the above cases corresponds to the response of a second-order system with a damping ratio of 3 and natural frequency of 2 rad/s?

[A]Case1 [B]Case2 [C]Case3 [D]Case4